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Eikonal Approach to Scattering from Coated Perfect Conductors in a Spinor Formalism

RONALD ROCKMORE†‡

The RAND Corporation, Santa Monica, California 90406

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Abstract

The eikonal approach is applied to the problem of the scattering of electromagnetic waves from an excluded volume in the presence of a weak external potential. The scattering of electromagnetic waves is treated in the spinor formalism previously developed by the author. The excluded volume is eventually taken to be a perfectly conducting cone, the external potential a coating of thickness δ, with complex dielectric constant ϵ', and permeability μ (tacitly assumed equal to 1). It is shown that to order $(N-1)$, where $N=(\epsilon'\mu)^{1/2}$, the eikonal approach in the spinor formalism yields results equivalent to those obtained from the vector theory of Überall in the particular case of nose-on backscattering, using the eikonal function corresponding to 'straight-line propagation'.

1. *Introduction*

It was shown in an earlier paper (Rockmore, 1966) that the problem of the scattering of high-frequency electromagnetic waves by weak dielectric bodies [in the eikonal approximation due to Glauber (Baker, 1964)] could be handled rather elegantly in a spinor formulation of the electromagnetic field. Indeed a number of complications encountered in the application of Saxon–Schiff (Überall, 1962) theory to the vector wave equation for, say $\mathbf{E}$,

$$(\nabla^2+k^2)\mathbf{E}=\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot\mathbf{E}+(1-\mu\epsilon')k^2\mathbf{E}-\mu^{-1}\boldsymbol{\nabla}\mu\times(\boldsymbol{\nabla}\times\mathbf{E}), \qquad (1.1)$$

were easily avoided in the eikonal approximation to the corresponding spinor wave equation,

$$(p-k)\psi=kU\psi \qquad (1.2)$$

† Permanent address: Physics Department, Rutgers University, New Brunswick, New Jersey 08903.

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where

$$\psi = \begin{bmatrix} -i\mathbf{E} \\ \hline \mathbf{H} \end{bmatrix} \tag{1.3}$$

with

$$p = \rho_1 \, \mathbf{s}\cdot(-i\nabla) \tag{1.4}$$

and

$$U = -[1 - \tfrac{1}{2}(\epsilon' + \mu)] + \rho_3 \tfrac{1}{2}(\epsilon' - \mu) \tag{1.5}$$

using the notation of Rockmore (1966).

Since the Saxon–Schiff method has also been the basis for the development by Überall (1964) of an approximation method for calculating the diffraction of electromagnetic waves in a situation where perfect conductors and weak scatterers (whose complex dielectric constant ϵ' and permeability μ have magnitudes near unity) are present simultaneously, i.e. the problem of the scattering from coated perfect conductors, it may be of some interest to extend the spinor formalism to this case as well. For definiteness and in order to compare with the work of Überall (1964) we also treat the semi-infinite, perfectly conducting and uniformly coated cone. [Moreover, for simplicity, our discussion is limited to the eikonal function corresponding to 'straight-line propagation,' (Überall, 1964) and to nose-on backscattering.] As there are no exact results with which to compare as in Rockmore (1966), we will emphasize the formal aspects of the problem (Section 2) and be content to exhibit the equivalence of the two approaches to order $(N-1)$ (both in the amplitude and in the rapidly varying exponentials).

2. *Green's Theorem for the 'Excluded Volume' in the Spinor Formalism*

In order to treat the scattering of electromagnetic waves in the spinor formalism in the situation outlined above, we take for the eikonal Green's function,

$$\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}') = [p + k^{-1}(p^2 - \mathbf{p}^2 + k^2)](4\pi|\mathbf{r}-\mathbf{r}'|)^{-1} \times \exp\left[ik|\mathbf{r}-\mathbf{r}'| + i\int_0^{|\mathbf{r}-\mathbf{r}'|} ds\, k V\left(\mathbf{r}' + \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}s\right)\right] \tag{2.1}$$

where

$$V = \tfrac{1}{6}\mathrm{tr}\, U \tag{2.2}$$

$\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')$ satisfies the spinor equation,

$$\begin{aligned}(p-k)\,\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}') &= (\mathbf{p}^2 - k^2)\,F_k^{(+)}(\mathbf{r},\mathbf{r}') \\ &= [(\nabla S)^2 - k^2 - i|\mathbf{r}-\mathbf{r}'|^2\nabla\cdot(|\mathbf{r}-\mathbf{r}'|^{-2}\nabla S)]\,F_k^{(+)}(\mathbf{r},\mathbf{r}') \\ &\quad + \delta(\mathbf{r}-\mathbf{r}')\end{aligned} \tag{2.3}$$

with $F_k^{(+)}(\mathbf{r},\mathbf{r}')$ defined by

$$\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}') = [p + k^{-1}(p^2 - \mathbf{p}^2 + k^2)]\,F_k^{(+)}(\mathbf{r},\mathbf{r}') \tag{2.4}$$

and $S(\mathbf{r},\mathbf{r}')$ by

$$S(\mathbf{r},\mathbf{r}') = k|\mathbf{r}-\mathbf{r}'| + \int_0^{|\mathbf{r}-\mathbf{r}'|} ds k V\left(\mathbf{r}' + \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} s\right) \tag{2.5}$$

Since

$$\begin{aligned}(p-k)\,\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}') &= \{[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T (\overleftarrow{p}^T - k)\}^T \\ &= -\{[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T (\overleftarrow{p} + k)\}^T \end{aligned} \tag{2.6}$$

where T denotes matrix transpose, Green's theorem takes the form here,

$$\begin{aligned}&\int_{\mathscr{D}} d\mathbf{r}\{[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T (p-k)\,\psi_{k_0}^{(+)}(\mathbf{r}) + [\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T (\overleftarrow{p}+k)\,\psi_{k_0}^{(+)}(\mathbf{r})\} \\ &\qquad = \int_{\mathscr{D}} d\mathbf{r}[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T kU(\mathbf{r})\,\psi_{k_0}^{(+)}(\mathbf{r}) - \psi_{k_0}^{(+)}(\mathbf{r}') \\ &\quad - \int_{\mathscr{D}} d\mathbf{r}[(\nabla S)^2 - k^2 - i|\mathbf{r}-\mathbf{r}'|^2 \nabla\cdot(|\mathbf{r}-\mathbf{r}'|^{-2}\nabla S)]\,F_k^{(+)}(\mathbf{r},\mathbf{r}')\,\psi_{k_0}^{(+)}(\mathbf{r})\end{aligned} \tag{2.7}$$

and leads to the integral equation for the spinor wave function $\psi_{k_0}^{(+)}(\mathbf{r}')$,

$$\begin{aligned}\psi_{k_0}^{(+)}(\mathbf{r}') &= \int_{\mathscr{D}} d\mathbf{r}[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T kU(\mathbf{r})\,\psi_{k_0}^{(+)}(\mathbf{r}) \\ &\quad - \int_{\mathscr{D}} d\mathbf{r}[(\nabla S)^2 - k^2 - i|\mathbf{r}-\mathbf{r}'|^2 \nabla\cdot(|\mathbf{r}-\mathbf{r}'|^{-2}\nabla S)]\,F_k^{(+)}(\mathbf{r},\mathbf{r}')\,\psi_{k_0}^{(+)}(\mathbf{r}) \\ &\qquad + i \int_{S_\infty + \sum_{ij} S_{ij}} ds[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T n\psi_{k_0}^{(+)}(\mathbf{r})\end{aligned} \tag{2.8}$$

where it is already assumed that the domain $\mathscr{D}$ is divided into sub-domains $\mathscr{D}_i$ ($i = 0, 1, 2$) with discontinuous values of ϵ', μ as in Ref. 5. Then S_{ij} is that surface between $\mathscr{D}_i$ and $\mathscr{D}_j$ bounding $\mathscr{D}_i$, with unit normal $\hat{n}_i$ pointing into $\mathscr{D}_j$.

Some additional reduction of equation (2.8) is necessary before we can focus our attention on the specific problem of the nose-on backscattering from a semi-infinite, perfectly conducting cone (see Fig. 1) of half-opening angle θ, and thus make contact with the results of Überall (1964). Thus,

$$\begin{aligned}&i \int_{S_\infty} dS[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T n\psi_{k_0}^{(+)}(\mathbf{r}) \\ &\quad \to i \int_{S_\infty} dS[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T nu(\mathbf{k}_0)\exp(i\mathbf{k}_0\cdot\mathbf{r}) \\ &\quad \to -ik \int_{S_\infty} dS \frac{1}{4\pi r}\left[\exp\left(ikr + i\int_0^\infty ds k V(\mathbf{r}' + \hat{n}s)\right)\right](n - n^2) \\ &\quad\quad \times nu(\mathbf{k}_0)\exp(i\mathbf{k}_0\cdot\mathbf{r} - ik\hat{n}\cdot\mathbf{r}')\end{aligned} \tag{2.9}$$

where $u(\mathbf{k}_0) = [-i\mathbf{E}_0/\mathbf{H}_0]$ and it is natural to write,

$$\psi_{k_0}^{(+)}(r) \xrightarrow[r\to\infty]{} \left[\frac{-i\mathbf{E}_0}{\mathbf{H}_0}\right] \exp(i\mathbf{k}_0\cdot\mathbf{r}) + \left[\frac{-i\mathbf{E}_{sc}(\mathbf{k}_0,\mathbf{k})}{\mathbf{H}_{sc}(\mathbf{k}_0,\mathbf{k})}\right]\frac{\exp(ikr)}{r}$$

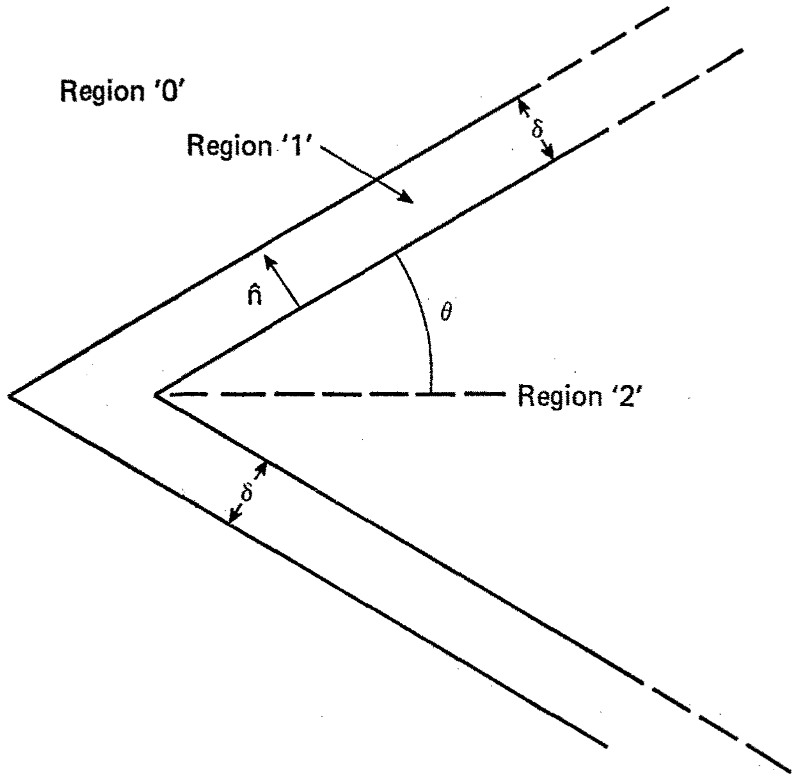

Figure 1.—The semi-infinite, perfectly conducting cone of half-opening angle θ (region '2'), with coating of thickness δ characterized by complex dielectric constant ϵ' and permeability μ (region '1'). The unit normal to the cone, $\hat{n}$, is also the unit normal, $\hat{n}_{21}$. Region '0' is free space ($\epsilon' = \mu = 1$).

Since $\not{k}_0 u(\mathbf{k}_0) = k\not{n}_0 u(\mathbf{k}_0) = ku(\mathbf{k}_0)$, the surface integral at infinity reduces to

$$i\int_{S_\infty} dS[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T \not{n}\psi_{k_0}^{(+)}(\mathbf{r}) \to \exp\left(i\int_0^\infty ds\,kV(\mathbf{r}'-\hat{n}_0 s)\right)u(\mathbf{k}_0)\exp(i\mathbf{k}_0\cdot\mathbf{r}') \tag{2.10}$$

In the case of the coated, perfectly conducting cone (Fig. 1), one has

$$\psi'_{sc} = \begin{bmatrix} -i\mathbf{E}'_{sc} \\ \mathbf{H}'_{sc} \end{bmatrix} \equiv \psi_{k_0}^{(+)}(\mathbf{r}') - u(\mathbf{k}_0)\exp(\mathbf{i}\mathbf{k}_0\cdot\mathbf{r}')$$

$$\rightarrow i \int_{S_{01}+S_{10}+S_{12}} dS\,[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T n\psi_{k_0}^{(+)}(\mathbf{r})$$

$$+ \int_{V_1} d\tau[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T kU(\mathbf{r})\,\psi_{k_0}^{(+)}(\mathbf{r})$$

$$+ \int_{V_1} d\tau F_k^{(+)}(\mathbf{r},\mathbf{r}')\,[(\nabla S)^2 - k^2 - i|\mathbf{r}-\mathbf{r}'|^2\nabla\cdot(|\mathbf{r}-\mathbf{r}'|^{-2}\nabla S)]\,\psi_{k_0}^{(+)}(\mathbf{r}) \tag{2.11}$$

and introducing one common unit normal $\hat{n}$ to all cone surfaces, assumed to point away from the body of the cone, one may write the contribution from the surface integrals of expression (2.11) as

Surface contribution

$$= -i \int_{S_{01}} dS\,[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T n[\psi_{k_0}^{(+)}(\mathbf{r})]_{\text{outside}}$$

$$+ \int_{S_{01}} dS\,[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T n[\psi_{k_0}^{(+)}(\mathbf{r})]_{\text{inside}}$$

$$- i \int_{S_{12}} dS\,[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T n[\psi_{k_0}^{(+)}(\mathbf{r})]_{\text{inside}} \tag{2.12}$$

For the reasons given earlier we content ourselves with the demonstration of the equivalence of the two eikonal approaches in this problem to order $(N-1)$ (both in the amplitude and the rapidly varying exponentials) for the case of nose-on backscattering $(\mathbf{k} = -\mathbf{k}_0)$. Überall's (1964) result, with which we are to compare, is

$$\mathbf{H}'_{bsc}(\infty) = -(4\pi r')^{-1}\exp(ikr')\Bigg\{-\int_{S_{01}} dS[\![(\hat{n}\times\mathbf{H})\times\nabla\exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]$$

$$+ \exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\,\hat{n}\times(\nabla\times\mathbf{H}) + k^{-2}[\hat{n}\times(\nabla\times\mathbf{H})]$$

$$\cdot\nabla\nabla\exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]]\!]_{\text{outside}} + \int_{S_{01}} dS[\![(\hat{n}\times\mathbf{H})$$

$$\times\nabla\exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0) + \exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\,\hat{n}\times(\nabla\times\mathbf{H})]\!]_{\text{inside}}$$

$$- \int_{S_{12}} dS(\hat{n}\times\mathbf{H})\times\nabla\exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]$$

$$+ \int_{V_1} d\tau\mathbf{H}\cdot\nabla\nabla\exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\Bigg\} \tag{2.13}$$

where we have dropped the term,

$$(4\pi r')^{-1}\exp(ikr')\int_{V_1} d\tau \exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\,\mathbf{H}[(K-k)^2+\exp(-i\delta_0)\nabla^2\exp(i\delta_0)]$$

with $K = kN = k(\epsilon'\mu)^{1/2}$, which is patently of order $(N-1)^2$.

$$\delta_0(\mathbf{r}) = \int_0^\infty [K(\mathbf{r}-\hat{n}_0 s)-k]\,ds = k(N-1)(z+\delta\csc\theta-\rho\cot\theta)$$

and

$$\nabla\delta_0 = -\hat{n}k(N-1)\csc\theta$$

[We add that it is tacitly assumed in Überall (1964) that $\mu = 1$ and we shall do so as well. Thus, for us,

$$V = -[1-\tfrac{1}{2}(\epsilon'+\mu)] = \tfrac{1}{2}(N^2-1)\approx N-1$$

to order $(N-1)$.] In comparing with the lower half of the spinor $\psi'_{bsc}(\infty)$, which is $\mathbf{H}'_{bsc}(\infty)$, we find the relations,

$$b\left[\frac{-i\mathbf{E}}{\mathbf{H}}\right] = \left[\frac{i\mathbf{b}\times\mathbf{H}}{\mathbf{b}\times\mathbf{E}}\right] \tag{2.14a}$$

$$ab\left[\frac{-i\mathbf{E}}{\mathbf{H}}\right] = \left[\frac{i\mathbf{a}\times(\mathbf{b}\times\mathbf{E})}{-\mathbf{a}\times(\mathbf{b}\times\mathbf{H})}\right] \tag{2.14b}$$

useful. We first consider the lower components of the volume integrals of our result, equation (2.11),

$$\text{Volume contribution} = J_1 + J_2 \tag{2.15a}$$

with

$$J_1 = \int_{V_1} d\tau[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T kU(\mathbf{r})\,\psi_{k_0}^{(+)}(\mathbf{r}) \tag{2.15b}$$

and

$$J_2 = \int_{V_1} d\tau F_k^{(+)}(\mathbf{r},\mathbf{r}')\,[(\nabla S)^2 - k^2 - i|\mathbf{r}-\mathbf{r}'|^2\nabla\cdot(|\mathbf{r}-\mathbf{r}'|^{-2}\nabla S)]\,\psi_{k_0}^{(+)}(\mathbf{r}) \tag{2.15c}$$

J_2 yields asymptotically

$$J_2 \xrightarrow[r'\to\infty]{} \frac{\exp(ikr')}{4\pi r'}\int_{V_1} d\tau\exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\,2k^2[1-\tfrac{1}{2}(\epsilon'+\mu)]\,[\psi_{k_0}^{(+)}(\mathbf{r})]_{\text{inside}} \tag{2.16}$$

with lower component,

$$(J_2)_{\text{lower comp.}} \to \frac{\exp(ikr')}{4\pi r'}\int_{V_1} d\tau\exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\,k^2(1-\epsilon')\,\mathbf{H}' \tag{2.17}$$

where $\mathbf{E}'$ and $\mathbf{H}'$ are values of $\mathbf{E}$ and $\mathbf{H}$ inside the coating. J_1 is asymptotically given by

$$J_1 \xrightarrow[r'\to\infty]{} \frac{\exp(ikr')}{4\pi r'} \int_{V_1} d\tau \exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\left[i\overleftarrow{\nabla} + \frac{1}{k}(-\overleftarrow{\nabla}^2 + \overleftarrow{\nabla}^2 + k^2)\right]$$
$$\times k\{-[1-\tfrac{1}{2}(\epsilon'+\mu)] + \rho_3\tfrac{1}{2}(\epsilon'-\mu)\}\,[\psi_{k_0}^{(+)}(\mathbf{r})]_{\text{inside}} \quad (2.18)$$

which simplifies to

$$J_1 \to \frac{\exp(ikr')}{4\pi r'} \int_{V_1} d\tau \exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\, i\overleftarrow{\nabla} k(\epsilon'-1)\begin{bmatrix}-i\mathbf{E}'\\ 0\end{bmatrix} \quad (2.19)$$

on setting $\mu = 1$ explicitly. Additional manipulation involving Maxwell's equations yields

$$J_1 \to \frac{\exp(ikr')}{4\pi r'}\left(-\int_{V_1} d\tau \exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\,k^2(1-\epsilon')\,\mathbf{H}'\right.$$
$$\left.+\int_{S_{01}} dS \exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\,k(\epsilon'-1)\,i\hat{n}\times\mathbf{E}'\right) \quad (2.20)$$

so that

$$J_1 + J_2 \to \frac{\exp(ikr')}{4\pi r'} \int_{S_{01}} dS \exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\,k(\epsilon'-1)\,i\hat{n}\times\mathbf{E}' \quad (2.21)$$

Next, it is easy to show that the lower components of the surface integral over S_{12},

$$L_3 \xrightarrow[r'\to\infty]{} \frac{i\exp(ikr')}{4\pi r'} \int_{S_{12}} dS\{K_0 + kn\csc\theta[1-\tfrac{1}{2}(\epsilon'+\mu)]\}\exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]$$
$$\times\, n[\psi_{k_0}^{(+)}(\mathbf{r})]_{\text{inside}} \quad (2.22)$$

give

$$(L_3)_{\text{lower comp.}} \to \frac{\exp(ikr')}{4\pi r'} \int_{S_{12}} dS\{(\hat{n}\times\mathbf{H})\times\boldsymbol{\nabla}\exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\} \quad (2.23)$$

The other surface integrals,

$$L_1 + L_2 = -i\int_{S_{01}} dS\,[\mathscr{G}_k^{(+)}(\mathbf{r},\mathbf{r}')]^T n\{[\psi_{k_0}^{(+)}(\mathbf{r})]_{\text{outside}} - [\psi_{k_0}^{(+)}(\mathbf{r})]_{\text{inside}}\} \quad (2.24)$$

yield asymptotically,

$$L_1 + L_2 \xrightarrow[r'\to\infty]{} \frac{i\exp(ikr')}{4\pi r'} \int_{S_{01}} dS \exp[i(\mathbf{k}_0\cdot\mathbf{r}+\delta_0)]\{-kn\csc\theta[1-\tfrac{1}{2}(\epsilon'+\mu)]$$
$$+ (K_0 n + nK_0)\csc\theta[1-\tfrac{1}{2}(\epsilon'+\mu)] - 2\hat{n}\cdot\mathbf{k}_0\csc\theta[1-\tfrac{1}{2}(\epsilon'+\mu)]\}$$
$$\times\, n[\psi_{k_0}^{(+)}(\mathbf{r})]_{\text{inside}} \quad (2.25)$$

with lower components,

$$(L_1 + L_2)_{\text{lower comp.}} \to \frac{i \exp(ikr')}{4\pi r'} \int_{S_{01}} dS\{-(\hat{n} \times \mathbf{H}') \times \boldsymbol{\nabla} - i[\hat{n} \times (\boldsymbol{\nabla} \times \mathbf{H})] \cdot \hat{n}_0 \boldsymbol{\nabla}\delta_0\} \exp[i(\mathbf{k}_0 \cdot \mathbf{r} + \delta_0)] \quad (2.26)$$

Agreement between the two approaches to order $(N-1)$ is then finally attained by applying Gauss' theorem and Maxwell's equations to Überall's (1964) volume integral

$$-(4\pi r')^{-1} \exp(ikr') \int_{V_1} d\tau \mathbf{H} \cdot \boldsymbol{\nabla\nabla} \exp[i(\mathbf{k}_0 \cdot \mathbf{r} + \delta_0)$$

transforming it into

$$-(4\pi r')^{-1} \exp(ikr') \int_{S_{01}} dS\hat{n} \cdot [(\boldsymbol{\nabla} \times \mathbf{H}) \times \boldsymbol{\nabla\nabla}] k^{-2} \exp[i(\mathbf{k}_0 \cdot \mathbf{r} + \delta_0)]$$

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